Journal of Econometrics 192 (2016) 40–54

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

# Efficiency of thin and thick markets[☆]

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ARTICLE INFO

*Article history:*
Received 1 March 2010
Received in revised form
10 October 2015
Accepted 23 October 2015
Available online 28 November 2015

*JEL classification:*
J6
J4

*Keywords:*
Thin and thick market
Matching function
Microfoundation
Market efficiency
Empirical application

ABSTRACT

In this paper, we propose a matching model to study the efficiency of thin and thick markets. Our model shows that the probabilities of matches in a thin market are significantly lower than those in a thick market. When applying our results to a job search model, it implies that, if the ratio of job candidates to job openings remains (roughly) a constant, the probability that a person can find a job is higher in a thick market than in a thin market. We apply our matching model to the U.S. academic market for new PhD economists. Consistent with the prediction of our model, a field of specialization with more job openings and more candidates has a higher probability of matching.



## 1. Introduction

In this paper, we are interested in the following question: Compare two markets, one of which has five candidates and five openings in five firms (each firm has one opening), and the other of which has fifty candidates and fifty openings. Which market has a lower average unemployment rate or a higher probability of successful match? The market with a lower unemployment rate is said to be more efficient than the one with a higher unemployment rate.

Intuitively, one would probably expect a higher matching probability in a thicker market. Indeed, this has been a common assumption in the literature. For example, Diamond (1982) assumes an increase in trading partners makes trade easier. He refers this assumption as "plausible". Howitt and McAfee (1988) show multiple externalities (one positive and one negative) may exist as market size differ. However, despite its intuitive appeal, literature provides no consensus on this point. For example, a thicker market has an adverse effect in Burdett et al. (2001), has no effect in Lagos (2000), and has a positive effect in Coles and Smith (1998). The different conclusions of these papers result from different assumptions on matching mechanism/cost. Burdett et al. (2001) use an urn-ball game to model a seller–buyer market over an homogeneous good. If search cost is very high so that each buyer can only afford to visit one seller, then urn-ball framework can be used to model the search behavior of buyers/sellers over a homogeneous good. However, in labor market, candidates typically conduct multiple searches and exhibit sufficient heterogeneity in the sense that they have different productivities/abilities and that firms have different (minimum) requirements. Therefore, the urn-ball framework is not appropriate for modeling a job-matching market.

In this paper, we propose a model that explains the observed phenomenon in the U.S. economics PhD matching market. In our model, firms are heterogeneous in types, and job candidates have heterogeneous productivities. A match between a firm and a candidate is assortative such that the firm is willing to hire any candidate with a productivity higher than its type, but prefers

☆ We thank four referees and a co-editor for their insightful comments that greatly improve our paper. We would also like to thank Zheng Li, Insik Min, Rich Prisinzano, and Jingyuan Yin for their research assistance. Discussions with Steve Bronars, Dean Corbae, Don Fullerton, Dan Hamermesh, Preston McAfee, Gerald Oettinger, Alvin Roth, Steve Trejo, and Randy Wright were very helpful. Qi Li thanks National Nature Science Foundation of China (Key Project Grant # 71133001) for partial research support on this project. All remaining errors are ours.

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http://dx.doi.org/10.1016/j.jeconom.2015.10.012
0304-4076/

the candidate with a higher productivity. A candidate is willing to accept any offer but prefers a higher-type firm. Further, the types and productivities are randomly drawn from a common distribution, and thereupon to become public information.

For such a market, we prove: (1) the matching probability does not depend on the underlying distribution that types and productivities are drawn upon. (2) When the numbers of firms and candidates increase or the market becomes thicker, the unemployment rate decreases or the matching probability increases. As the number of firms and candidates approaches to infinity, the matching probability approaches to 1. (3) The conclusion that a thicker market has a larger matching probability than a thinner market continues to hold for the following four more general cases: first, the number of openings does not equal the number of candidates; second, the firm type and the productivity of a candidate are random draws from different distributions; third, the firms and candidates arrive at the market sequentially; and finally, candidates have reservation wage and will not take offers below their reservation wage.

Information on the U.S. academic market for new PhD economists is used to check the validity of our model. We gather the American academic job openings listed for each field in the September, October, November and December issues of Job Openings for Economists (JOE), in both 1999 and 2000. In the following year we find out how many of these openings are filled. The ratio of the total number of filled jobs divided by the openings in each field is the measure of the probability of job matching. CVs of all job candidates from the top 50 departments of economics in the U.S. universities in year 2000 and 2001 are also collected from each university's placement website or candidates own websites. The empirical estimates support our theoretical hypothesis: a thicker market does have a higher matching probability than a thinner market. In particular, in the new PhD markets in economics, when the numbers of openings and candidates are five in one field, the matching probability is 0.361. When the numbers of openings and candidates are fifty, the matching probability is 0.523.

Previous literature often uses changes in unmatched probabilities such as vacancy rates or unemployment rates to empirically estimate matching functions.[1] Using the market data for PhD economists offers several advantages over regular job markets. First, there is less of an information problem in this market in the sense that each institute receives applications from almost all potentially qualified job candidates, and almost all job openings are well known to all candidates, as they are published in a single magazine JOE. Second, there is a reasonable consensus in terms of the ranking of a job, i.e., a job in a better-ranked department is considered by most to be a better job. Third, there is some consensus in terms of the ranking of candidates, although significant heterogeneity still exists.

The effect of "thickness" in the market has been studied extensively in the microstructure literature in finance under the term "liquidity". For example, in Lippman and McCall (1986), a thicker market indicates that more transactions of a homogeneous good take place in a unit of time. In their paper, liquidity is defined in terms of the time elapsed between transactions. This length of time is a function of a number of factors, including the frequency of offers and the flexibility of prices, among others. In an empirical study on common factors that affect liquidity, Chordia et al. (2000) use five liquidity measures including the difference in prices offered by buyers and sellers, and in quantities offered by buyers and sellers in a period of time. In their approach, the smaller the difference between the prices and the larger the quantities offered by the buyers and sellers of a homogeneous good (an equity), the more liquid a market is. One distinguishing feature in financial markets is that buyers and sellers often arise endogenously. If prices are low, potential sellers easily become buyers. In the labor market, it is hard for workers to become employers or vice versa. Therefore, the pool of employers and employees is often exogenously determined.

Since our model relates the matching probability with the thickness of the market, it provides a matching function with a microfoundation. The importance of the matching function has been discussed in a survey paper by Petrongolo and Pissarides (2001) who argue that both the matching function and the demand-for-money function are as important as the production function as a tool kit for macroeconomists. Petrongolo and Pissarides further state that (page 425) "Currently, the most popular functional form, Cobb–Douglas with constant returns to scale, is driven by its empirical success and lacks microfoundations. The most popular microeconomic models, such as the urn-ball game, do not perform as well empirically". Our paper provides a matching function with microfoundations and the matching function is shown to perform reasonably well for the empirical data we collected.

The rest of the paper is organized as follows: Section 2 introduces the matching model and the basic implications of the model. Section 3 presents the empirical application of the model using the data collected from the U.S. academic job market for new PhD economists. Section 4 concludes the paper.

## 2. The model

### 2.1. The matching mechanism

Let $u$ be a measure of the productivity of a job candidate, and $v$ be a firm's type. The match between the firm and the candidate is assortative, with a production function given by $f(u, v) = uv$, as suggested in Lu and McAfee (1996). The cost of hiring a worker consists of wage and other costs, such as the capital investment that the firm is willing to make for this opening. Both the wage and capital investment are assumed to be proportional to the firm type $v$. The total cost is normalized to be $v^2$. The firm's profit function is written as:

$$\pi(u, v) = \max\{0, uv - v^2\}. \quad (1)$$

In this simple model, the firm will hire a candidate if $u \geq v$, and it prefers a candidate with a higher productivity than one with a lower productivity.

Without loss of generality we assume that $v$ takes value in $[0, C]$ for some $C > 0$. We assume a candidate's utility function is:

$$w(u, v) = \max\{\delta u, v\}. \quad (2)$$

The candidate is willing to accept all job offers as long as $v \geq \delta u$ but prefers the firm with higher $v$ than one with a lower $v$. Here we let the candidate's reservation wage $\delta u$ be dependent on his/her ability index $u$. This is reasonable because candidates with high productivities should have better outside opportunities, such as better-paid non-academic jobs, than those with lower abilities. Eqs. (1) and (2) present necessary conditions for a match:

$$u \geq v \geq \delta u.$$

To simplify the theoretical analysis in this paper we will mainly consider a benchmark case with $\delta = 0$, then (2) becomes $w(u, v) = \max\{0, v\} = v$ because $v \geq 0$. In this case individuals will take any offer, but firm will only hire a worker if $u \geq v$. Therefore, the firm type $v$ may also be thought as the firm's minimum quality requirement.

The matching technology between firms and candidates defined in (1) and (2) is similar to the matching mechanism between hospitals and medical interns or residents as described in Roth (1982). In Roth (1982), almost all medical interns would find jobs because of a large demand for their services. Therefore, Roth (1982)

[1] See, for example, Blanchard and Diamond (1989) and Berman (1997).

and the subsequent literature do not put much emphasis on matching probabilities.[2]

Our primary goal here is to examine how the matching probability varies with the number of job candidates and the number of openings. We consider the problem of $V$ firms and $U$ job candidates. Let the firms' types be $v_1, \ldots, v_V$, and job candidates' productivities be $u_1, \ldots, u_U$. We assume that all productivities and job types are randomly drawn from a common continuous distribution $F(\cdot)$ so that no two productivities are exactly the same with probability 1 as any job types. All candidates $u_i$'s and all firm types $v_j$'s are assumed to be known after they are drawn.

According to our model described by Eqs. (1) and (2), a candidate $i$ may be hired by firm $j$ only if the productivity of the applicant $u_i$ is higher than the type of the firm $v_j$. If more than one applicant has a higher productivity than the firm type $v_j$, firm $j$ hires the candidate with the highest productivity. Similarly, if more than one firm has lower types than applicant $i$'s productivity $u_i$, the applicant prefers the firm with the highest type. After a match occurs, both the firm and the candidate are out of the market. The process continues until no candidate has a higher productivity than any of the remaining firm types.

In the academic market, a better department is preferred by all new PhDs. The constraint in the market is that each department is different in its type and a better department has a higher $v$. Each department prefers the candidate with the highest quality that satisfies their type. In this case, a necessary condition for a trade to occur between candidate $i$ and department $j$ is $u_i \geq v_j$.

In this matching mechanism, the job candidate who has the highest productivity matches with the firm with the highest type, provided that this highest ranked candidate meets the type of the firm. Otherwise, the firm leaves the market without filling its opening. However, the applicant who does not match with the highest-type firm has additional chances to match with other firms. This matching process repeats in the remaining pool of the applicants and firms.

An alternative way to describe our matching technology is as follows. First we sort all the randomly drawn productivities and types. Then a job candidate with the highest productivity matches with the firm with the highest type, as long as the productivity is higher than the type. If a match occurs, both the candidate and the firm leave the market. This process is repeated until no firm's types are lower than any remaining candidates.

Consider order statistics $v_{(1)} < v_{(2)} < \cdots < v_{(V)}$ and $u_{(1)} < u_{(2)} < \cdots < u_{(U)}$ that are obtained from $v_1, \ldots, v_V$ and $u_1, \ldots, u_U$, respectively. We are interested in the probability that a randomly chosen candidate can find a job. Let $r$ be the number of people that find jobs, $0 \leq r \leq n = min\{V, U\}$. Let $Pr(r)$ be the probability that exactly $r$ candidates find jobs. The average or expected value of $r$ is given by:

$$M_{U,V} \stackrel{def}{=} E(r) = \sum_{r=0}^{n} r\,\Pr(r) = \sum_{r=1}^{n} r\,\Pr(r). \tag{3}$$

In the following sections, we study how the matching probabilities vary with the number of vacancies and the number of candidates. We first discuss the case where the number of vacancies and the number of candidates are the same, and then we proceed with the case where they are different.

[2] Niederle and Roth (2003) study the matching probability of the gastroenterologists market. They find matching probability increases when the market becomes thicker (through a centralized clearinghouse).

### 2.2. When the number of openings equals the number of candidates

Our primary interest in this paper is to study how the matching probability changes when the number of openings and the number of candidates change. Our discussion starts with the case where the number of applicants is the same as the number of openings. Let $n = V = U$, and we write $M_{n,n} = M_n$. We denote by $A_n$ the probability that a randomly selected person can find a job (when $V = U$), i.e.,

$$A_n = \frac{1}{n} M_n = \frac{1}{n} \sum_{r=1}^{n} r\,\Pr(r).$$

We investigate below how $A_n$ changes as $n$ changes. We build our model from the simplest case where there is one firm and one job candidate.

**The case of $n = 1$:**

Let $u$ and $v$ be randomly drawn from the same distribution $F(\cdot)$ and with a probability density function $f(\cdot)$. A match occurs if and only if $u \geq v$.

$$A_1 = \int_{\{v<u\}} f(v)f(u)dvdu = \int_{-\infty}^{\infty} \left[\int_a^u dF(v)\right] dF(u)$$
$$= \int_{-\infty}^{\infty} F(u)dF(u) = 1/2.$$

In the simple case of one applicant and one job opening, given that both $u$ and $v$ are randomly drawn from the same distribution, the probability that one random draw is larger than the other is 1/2.

**The case of $n = 2$:**

Let $u_1, u_2$ be random draws of the two candidates' productivities, and let $v_1, v_2$ be random draws of the minimum requirements of two job openings. All are from the same distribution. Let $u_{(1)} < u_{(2)}$ be the order statistic of $u_1, u_2$ and $v_{(1)} < v_{(2)}$ be the order statistic of $v_1, v_2$. Using Lemma A.1 given in Appendix A we have:

$$\begin{aligned}
\Pr(0) &= \Pr(u_{(1)} < u_{(2)} < v_{(1)} < v_{(2)}) \\
&= (2!)^2(1/4!) = 1/6. \\
\Pr(2) &= \Pr(u_{(2)} > v_{(2)}, u_{(1)} > v_{(1)}) \\
&= \Pr(u_{(2)} > v_{(2)} > u_{(1)} > v_{(1)}) \\
&\quad + \Pr(u_{(2)} > u_{(1)} > v_{(2)} > v_{(1)}) \\
&= 2\{(2!)^2(1/4!)\} = 1/3. \\
\Pr(1) &= 1 - Pr(0) - Pr(2) \\
&= 1 - (1/6) - (1/3) = 1/2.
\end{aligned}$$

Therefore we have

$$A_2 = \frac{1}{2} \sum_{r=1}^{2} r\,\Pr(r) = [(1/2) + 2(1/3)]/2 = 7/12.$$

We observe that $A_2 = 7/12 > 1/2 = A_1$. That is, when the market becomes thicker ($n$ increases from 1 to 2), the probability that *each* person can find a job is increased from 1/2 to 7/12. To understand the intuition of this result, note that since $\{u_1, u_2\}$ and $\{v_1, v_2\}$ are from the same distribution, the order statistics also have the distribution: $F_{u_{(1)}}(\cdot) = F_{v_{(1)}}(\cdot)$ and $F_{u_{(2)}}(\cdot) = F_{v_{(2)}}(\cdot)$. Given this, we have:

$$\Pr(u_{(1)} > v_{(1)}) = 1/2, \quad \text{and} \quad \Pr(u_{(2)} > v_{(2)}) = 1/2. \tag{4}$$

If (4) were the only cases that candidates and openings match, we would still end up with a matching probability of 1/2. However, an additional chance exists even when $u_{(1)} < v_{(1)}$ and $u_{(2)} < v_{(2)}$ since it is still possible to have $u_{(2)} > v_{(1)}$. This additional chance of matching is the source of the effect of a thicker market.

**Table 1**
Matching probabilities based on 100,000 simulations ($n = V = U$). (Openings and candidates have the same distributions.)

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $A_n$ | 0.5003 | 0.5835 | 0.6337 | 0.6682 | 0.6940 | 0.7135 | 0.7288 | 0.7448 | 0.7562 | 0.7665 |
| Std of $A_n$ | 0.5000 | 0.3434 | 0.2765 | 0.2379 | 0.2116 | 0.1932 | 0.1786 | 0.1661 | 0.1561 | 0.1481 |
| $n$ | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 1000 |
| $A_n$ | 0.8258 | 8543 | 0.8720 | 0.8845 | 0.8938 | 0.9013 | 0.9074 | 0.9124 | 0.9163 | 0.9725 |
| Std of $A_n$ | 0.1041 | 0.0853 | 0.0734 | 0.0657 | 0.0599 | 0.0554 | 0.0523 | 0.0494 | 0.0459 | 0.0144 |

In Appendices A.3 and A.4, we calculate the matching probabilities for $n = 3$ and $n = 4$. Although the same approach can be applied to compute $A_n$ for any $n > 4$, computation is increasingly burdensome and tedious as $n$ increases. A simple alternative is to use simulations to numerically compute $A_n$. Let $A_{n,j}$ be the estimated value of $A_n$ based on the $j$th simulation draw of $(u_1, \ldots, u_n)$ and $(v_1, \ldots, v_n)$, i.e., $A_{n,j}$ equals the number of people finding jobs in the $j$th random draw ($j = 1, \ldots, J$). We estimate $A_n$ by $\bar{A}_{n.} = J^{-1}\sum_{j=1}^{J} A_{n,j}$. Provided that $J$ is sufficiently large, we can obtain an estimated value of $A_n$ with any desired accuracy. We use $J = 100{,}000$ in our simulation. We also compute the sample standard error of $\{A_{n,j}\}_{j=1}^{n}$ by $[(J-1)^{-1}\sum_{j=1}^{J}(A_{n,j} - \bar{A}_{n.})^2]^{1/2}$. The results are given in Table 1.

We have already shown that $A_1 = 0.5, A_2 = 7/12 \approx 0.5833$. In the Appendix we also compute the exact values of $A_n$ for $n = 3,\ 4$; they are $A_3 = 19/30 \approx 0.6333$ and $A_4 = 187/280 \approx 0.6679$. Comparing these results with the simulation results of Table 1, we see that the simulation results differ from the theoretical results only in the fourth decimal.

From Table 1 we observe that $A_n$ increases as $n$ increases, while the standard error decreases as $n$ increases. The monotonically increasing relationship between matching probabilities and the thickness of the market can also be clearly seen in Fig. 1. The solid line in Fig. 1 illustrates the matching probabilities as a function of the number of candidates. As $n \to \infty$, both the candidates and openings become dense in the support of $f(\cdot)$. Therefore, the probability of matching is expected to converge to one as $n \to \infty$. This is indeed the case as the next lemma shows.

**Lemma 1.** *The employment rate or the matching probability $A_n$ converges to one as $n \to \infty$.*

The proof of Lemma 1 is given in the Appendix. Note that Lemma 1 does not mean that as $n \to \infty$, every individual will find a match. In fact the total number of unmatched candidates, calculated by $n(1 - A_n)$, also goes up as $n$ increases. For example, when $n = 10$, 100 and 1000, the average numbers of unemployed workers are roughly 2, 8, and 30, respectively. Therefore, the corresponding percentages of unemployed are 20%, 8% and 3%, respectively. It is, $1 - A_n$, the unemployment rate that goes down as $n$ increases.

Our theoretical analysis and simulation results show that: (1) A thicker market provides a larger chance of matching; (2) the probability of matching varies less in a thicker market than in a thinner market. (3) The matching probability is an increasing and concave function in market size. Matching probability exhibits increasing return to scale in marker size. We derive our matching function with a microfoundation. This is in contrast to urn ball-game based matching model which produces a decreasing return to scale matching function and does not describe the real world matching behavior well, and the commonly used Cobb–Douglas constant return to scale matching function which does not have microfoundations.

We would like to mention that in our matching model, non-matches occur when all firms are not able to find workers that are above their quality threshold. This is more likely to happen in a thin market than in a thick market. Up to now we have only considered

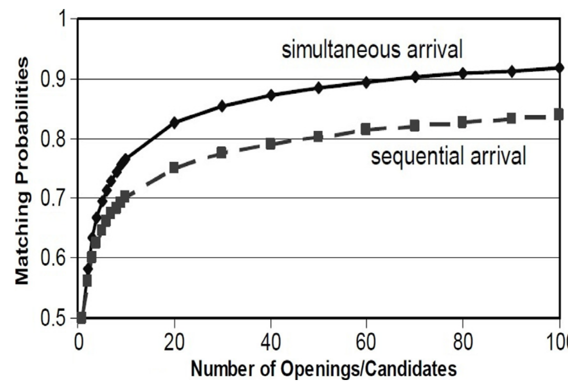


**Fig. 1.** Matching probabilities as a function of thickness.

the case that $u$'s and $v$'s are drawn from a common distribution. We explore the case that they are drawn from different distributions at the next section.

### *2.3. Candidates and openings are drawn from different distributions*

Previous results are obtained by assuming that the types of firms and candidates' productivities have the same distribution. Next, we briefly discuss the case that they have different distributions. We show that in this case the matching probability will depend on the specific distribution functions, but a thicker market still has a larger probability of matching.

We first consider a simple case where candidates are randomly drawn from uniform[0, 1], and the openings are randomly drawn from uniform $[\delta, 1+\delta]$, $0 \le \delta \le 1$. We will only consider the case of $V = U = n$. In Appendix A.5 we show that:

$$A_1 = \frac{1}{2}(1-\delta)^2, \quad \text{and}$$
$$A_2 = \frac{7}{12}(1-\delta)^2 + \frac{1}{12}\delta(1-\delta)^2(2+3\delta). \tag{5}$$

Obviously, $A_2 > A_1$ for all $\delta \in [0, 1]$. A thicker market still has a larger probability of matching. For $n > 2$, the computation becomes quite tedious. However, one can use simulations to compute $A_n$ easily for any value of $n$. Fig. 2 illustrates how the simulated matching probabilities vary with $n$ and with $\delta$. Two patterns emerge from Fig. 2. First, as expected, a larger difference in means results in lower matching probabilities. Second, for a fixed value of $\delta$, the matching probabilities increase as the market becomes thicker. By exactly the same argument as in the proof of Lemma 1, one can show that as $n \to \infty, A_n \to 1 - \delta\ (0 < \delta < 1)$.

Note that when $\delta \ge 1$, $A_n = 0$ for all $n$, because in this case the highest candidate's productivity is lower than the lowest firm's type. However, if the two distributions are $N(\mu, \sigma^2)$ and $N(\mu + \delta, \sigma^2)$ where the two means also differ by $\delta$, then $A_n > 0$ for all values of $\delta$. This simple example shows that when the two distributions are different, the matching probability will depend on the specific distributions.

Below we consider the case that $u$'s and $v$'s are drawn from two general distributions (not necessarily from uniform distributions).

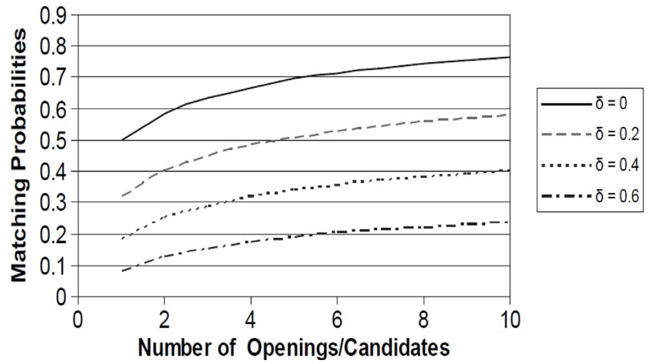


**Fig. 2.** Matching probabilities openings are from uniform[0, 1] and candidates are from uniform[$\delta$, $1+\delta$].

Suppose that $u$'s are drawn from distribution $F$, and $v$'s are drawn from distribution $G$ with $F \neq G$. For $n=1$, we have

$$A_1 = E(r) = P(1) = P(v_1 < u_1) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{u_1} dG(v_1)\right] dF(u_1)$$
$$= \int G(u_1)dF(u_1) = \int G(u)dF(u).$$

For $n=2$ in Appendix A we show that

$$A_2 = \int \left\{2G(u)F(u) - 2G^2(u)F(u) + G^2(u) + 2\left[G(u)\int_{-\infty}^{u} G(s)dF(s) - \int_{-\infty}^{u} G^2(s)dF(s)\right]\right\} dF(u).$$

It is easy to check that when $F(\cdot) = G(\cdot)$, we get $A_2 = \int \left[3F^2(u) - \frac{5}{3}F^3(u)\right] dF(u) = 7/12$ as it should. From the above derivations of $A_1$ and $A_2$ we know that $A_2 > A_1$ if and only if

$$\int G(u)[1-G(u)][2F(u)-1]dF(u) + 2\int\left[G(u)\int_{-\infty}^{u} G(s)dF(s) - \int_{-\infty}^{u} G^2(s)dF(s)\right] dF(u) > 0. \tag{6}$$

It is easy to see that when $u$ and $v$ are continuous random variables with overlapping supports, the second term on the left hand side of the above inequality is positive. Hence, a sufficient condition for $A_2 > A_1$ is

$$\int G(u)[1-G(u)][2F(u)-1]dF(u) \geq 0.$$

It is easy to verify that for the special case $F(\cdot) = G(\cdot)$ the sufficient condition indeed holds, i.e., $\int F(u)[1-F(u)][2F(u)-1]dF(u) = 0$. Also when $u$ and $v$ are symmetrically distributed around zero, the above sufficient condition holds as well. For some skewed distribution, the above sufficient condition may not hold for some values of skewness parameters. However, even when the sufficient condition is not satisfied. We still have $A_2 > A_1$ because the second term of (6) is positive and large than the absolute value of the first term. Due to complexity of the problem, we are unable to prove that $A_n$ is monotone in $n$ for all $F$ and $G$. Below we use simulations to compute $A_n$ for 10 different pairs of $(F, G)$ for $n = 1, \ldots, 20$. The first case is $F = G$ (a uniform[0, 1] distribution) that serves a benchmark case, the other nine cases all have $F \neq G$. Specifically, Table 2 reports the simulation results when $U$ and $V$ are in different ranges or follow different distributions. There are 10 cases altogether.

1. $U \sim Uniform[0, 1], V \sim Uniform[0, 1]$;
2. $U \sim Uniform[0, 1], V \sim Uniform[0, 2]$;
3. $U \sim Uniform[0, 2], V \sim Uniform[0, 1]$;
4. $U \sim Uniform[0.5, 1.5], V \sim Uniform[0, 2]$;
5. $U \sim N(0, 1), V \sim N(0, 2)$;
6. $U \sim N(0, 2), V \sim N(0, 1)$;
7. $U \sim Uniform[0, 1], V \sim N(0, 1)$;
8. $U \sim N(0, 2), V \sim Uniform[0, 1]$;
9. $U \sim \chi_1^2, V \sim \chi_2^2$;
10. $U \sim \chi_2^2, V \sim \chi_1^2$.

The results are given in Table 2 where one can see that $A_n$ indeed increases with $n$, suggesting that $A_n$ is monotone in $n$ for general distributions $F$ and $G$. Therefore, it is quite natural to conjecture that even when $F \neq G$, the matching probabilities $A_n$ increase with $n$.

We would like to emphasize that due to the complexity of the problem, in this paper we are not able to prove that a thick market is more efficient than a thin market for the general case with $u$'s and $v$'s are drawn from different distributions. We rely on intuitions and simulations (as supporting evidences) to conjecture that the conclusion that a thick market is more efficient than a thin market holds true for general distributions. Theoretically verify this conjecture is beyond authors' technicality.

### 2.4. The number of firms is different from the number of candidates

In the previous section, we only focus on the case where the number of candidates equals the number of openings. In a real market, it is unlikely that there will be exactly the same number of candidates and openings. In this section, we consider cases where the number of candidates is different from the number of openings. They are still random draws from a common distribution.

Let $U$ be the number of candidates and $V$ be the number of openings. The number of people who find jobs, $r$, must satisfy $0 \leq r \leq n = min\{U, V\}$. Recall that the expected value of $r$ is:

$$M_{U,V} = E(r) = \sum_{r=0}^{n} r \Pr(r). \tag{7}$$

Here, we summarize some properties of the matching function. (i) $M_{U,V} = M_{V,U}$ is symmetric in $V$ and $U$, (ii) $M_{U,V}$ increases as either $V$ or $U$ increases, (iii) if both $V$ and $U$ increase with $V/U = c$, where $c$ is a fixed positive constant, then $B_{U,V} = M_{U,V}/V$ increases as $V$ ($U = V/c$) increases.

Property (i) follows from a simple symmetry argument. (ii) is true because adding more candidates or openings to a market obviously cannot reduce the number of matching; in fact, there is a positive probability of increasing the number of matching, thus the average matching of $M_{U,V}$ will be larger. (iii) is the most interesting result: it says that when the market becomes thicker, the probability of matching success increases for both candidates and openings. The intuition behind (iii) is quite simple. We have already seen that this is true for the case of $V = U = n$. In Appendix A.6 we show how to compute $M_{U,V}$ (or $B_{U,V}$) for the general $(U, V)$ case. For example, for $(U, V) = (1, 2)$ (or $(2, 1)$), $M_{U,V} = 2/3$; for $(U, V) = (1, 3)$, $M_{U,V} = 3/4$; and for $(U, V) = (2, 4)$, $M_{U,V} = 23/15$. First we note that $B_{1,2} = 2/3 < B_{2,4} = (1/2)(23/15) = 23/30$, so that as the number of $V$ and $U$ doubles (the market becomes thicker), the matching probability increases. Next we compare the case of $(U, V) = (1, 3)$ and $(2, 2)$, where we have $M_{2,2} = 7/6 > 3/4 = M_{1,3}$. With the same total number of openings and candidates, the closer the ratio of $V/U$ is to 1, the higher the averaging number of people that can find jobs.

Again, a simple alternative is to use simulations to estimate $M_{U,V}$ ($B_{U,V}$). Simulation results (not reported here to save space) show our finding that a thick market is more efficient than a thin market remains to be true when $U \neq V$. This is easy to understand because exactly the same explanation/intuition for the $U = V$ case carry through to the case of $U \neq V$. In the next section we will use the simulation method to help us evaluate some of our proposed matching functions.

**Table 2**
Openings and candidates have the different distributions ($n = V = U$).

| $n$ | $F$ 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.499 | 0.250 | 0.749 | 0.504 | 0.501 | 0.499 | 0.686 | 0.315 | 0.292 | 0.706 |
| 3 | 0.633 | 0.349 | 0.847 | 0.597 | 0.610 | 0.611 | 0.758 | 0.390 | 0.414 | 0.827 |
| 5 | 0.692 | 0.387 | 0.888 | 0.638 | 0.658 | 0.658 | 0.784 | 0.420 | 0.468 | 0.878 |
| 7 | 0.731 | 0.409 | 0.911 | 0.661 | 0.686 | 0.686 | 0.799 | 0.437 | 0.500 | 0.906 |
| 10 | 0.767 | 0.431 | 0.931 | 0.680 | 0.713 | 0.712 | 0.810 | 0.451 | 0.530 | 0.933 |
| 15 | 0.803 | 0.449 | 0.949 | 0.700 | 0.738 | 0.737 | 0.820 | 0.465 | 0.561 | 0.957 |
| 20 | 0.826 | 0.460 | 0.960 | 0.710 | 0.752 | 0.752 | 0.825 | 0.473 | 0.580 | 0.969 |

### 2.5. A matching function

Because our model relates the matching probability with the thickness of the market, it can provide a matching function with a microfoundation. A series of matching functions has already been introduced in the literature; here we briefly discuss some of the existing matching functions and compare them with our matching function.

In a typical matching model with constant return to scale, the thickness of the market does not enter the matching probability. The relationship between the number of people who are looking for jobs and the number of people who find jobs is different from our claim that market thickness has a positive effect on the job matching ratio. For example, consider a typical matching model with constant return to scale,

$$M = m(U, V) = V \cdot m\left(\frac{U}{V}, 1\right),$$

where $m(U, V)$ is the matching function, $M$ is the number of people who find jobs, $V$ and $U$ are numbers of job openings and job searchers. The second equality of the previous equation is due to the assumption of the constant return to scale. Rearranging the previous equation, we get:

$$B_{U,V} = \frac{M}{V} = m\left(\frac{U}{V}, 1\right) \tag{8}$$

where $B_{U,V}$ is firms' matching probability. If the ratio of candidates to openings is fixed, so is the matching probability $M/V$. A particular form of constant return to scale function is $M/V = 1 - \exp(-cU/V)$, which is used in Blanchard and Diamond (1994) where $c$ is the intensity of the search. Other interesting works related to our matching model include Burdett et al. (2001) and the stock-flow matching of Coles and Smith (1998).

It would be ideal if one could derive an explicit functional form to relate matching probabilities with the thickness of the market. While this goal may be quite difficult to accomplish, we are able to propose a parsimonious approximate matching function which satisfies some basic properties of the theoretical matching function. We will show that this approximate matching function can fit the theoretical matching probabilities very well. We are interested in obtaining a probability matching function, say $B_{U,V} = M_{U,V}/V$. However, it is easier to impose restrictions on the matching function $M_{U,V}$. We assume that the matching function $M(u, v)$ possesses the following properties:

(i) $M_{U,V}$ is symmetric on $(U, V)$.
(ii) For any finite values of $(U, V)$, $M_{U,V} \leq \min\{U, V\}$, and $M_{U,V}$ is an increasing function in $U$ ($V$) for a fixed value of $V$ ($U$).
(iii) Let $d = \sqrt{U^2 + V^2}$ denote the distance of $(U, V)$ to the origin. For $(U, V) \in R_+^2$ with $d = c$, where $c$ is a constant, $M_{U,V}$ is monotonically decreasing as $(U, V)$ moves away from the middle point of $V = U$ (along the arc of $d = c$).

The following simple matching function satisfies the above three conditions:

$$M_{U,V} = \alpha_0 + \alpha_1 \min\{U, V\} + \frac{\alpha_2}{d}, \tag{9}$$

where $d = \sqrt{V^2 + U^2}$, and $\alpha_0$, $\alpha_1$, and $\alpha_2$ are parameters ($\alpha_1$ is positive and $\alpha_2$ is negative).

It is obvious that $M_{U,V}^{(0)}$ in (9) satisfies properties (i) and (ii) above. To see that it also satisfies (iii), note that when $d = c$ is a constant,

$$M_{U,V} = \alpha_0 + \alpha_1 \min\{U, V\}|_{d=c} + \alpha_2/c,$$

which decreases monotonically as $(U, V)$ moves away from the middle point of $U = V$ (along the arc of $d = c$).

By rearranging (9) in terms of matching probability (and replace $\alpha_0/V$ by $\alpha_0$), we get

$$\frac{M_{U,V}}{V} = \alpha_0 + \alpha_1 \min\left\{\frac{U}{V}, 1\right\} + \frac{\alpha_2}{Vd}. \tag{10}$$

Using both simulated data and the empirical data we found that if we replace $\alpha_2/(Vd)$ by $\alpha_2/d$ (removing the $1/V$ factor) in (10), we can get better fit. Therefore, we also consider the following alternative approximate matching function:

$$\frac{M_{U,V}}{V} = \alpha_0 + \alpha_1 \min\left\{\frac{U}{V}, 1\right\} + \frac{\alpha_2}{d}. \tag{11}$$

Note that model (10) and model (11) have the same number of parameters so that we can compare the goodness-of-fit of these two models. As we mentioned above we find that (11) has a higher goodness-of-fit $R^2$ than (10) using both theoretical (simulated) matching probabilities and the empirical data. This implies that (11) is preferred to (10) in modeling a matching function.

A more flexible model than (11) is

$$\frac{M_{U,V}}{V} = \alpha_0 + \alpha_1 \min\left\{\frac{U}{V}, 1\right\} + \frac{\alpha_2}{d^{\alpha_3}}. \tag{12}$$

When $\alpha_3 = 1$, (12) reduces back to (11). To examine how well our proposed matching functions approximate the theoretical (simulated) matching function, we carry out a least squares regression, using (simulated) theoretical values of $B_{U,V} = M_{U,V}/V$ as the dependent variable, and estimate models (10)–(12). We consider three cases: (i) Both $u$ and $v$ are draws from uniform[0,1] distribution; (ii) $u \sim$ uniform[0, 1] and $v \sim$ uniform[0, 2]; (iii) $u \sim$ uniform[0, 2] and $v \sim$ uniform[0, 1]. We first consider sample size $n = 100$ ($1 \leq u, v \leq 10$). The regression results for estimating models in (10)–(12) for cases (i)–(iii) are reported in Tables 3–5.

As can be seen in Tables 3–5, our specifications can explain the (simulated) theoretical matching probability well, with $R^2$ being at least 0.917.

For the case of $F = G$ (both $u$ and $v$ are draws from uniform[0, 1]), Table 3 shows that the $R^2$ is 0.951 for model (11) and 0.957 for model (12), suggesting that model (12) fits the data slightly better than model (11). However, the $p$-value for testing

**Table 3**
Regressions results ($U$, $V$ both Uniform[0, 1]).

| Models | $1 \leq U, V \leq 10$ | | |
|---|---|---|---|
| | Model (10) | Model (11) | Model (12) |
| Constant | 0.015 (0.841)[a] | 0.075 (4.28) | 0.060 (1.96) |
| $\min\{U/V, 1\}$ | 0.805 (37.7) | 0.787 (42.8) | 0.779 (42.3) |
| $1/(U^2+V^2)^{1/2}$ | | −0.423 (−7.07) | −0.456 (−4.55) |
| $1/[V(U^2+V^2)^{1/2}]$ | −0.281 (−3.20) | | |
| $\alpha_3$ | | | 1.19 (3.05) |
| $p$-value for $H_0$: $\alpha_3 = 1$ | | | 0.626 |
| $R^2$ | 0.937 | 0.951 | 0.957 |
| Number of observations | 100 | 100 | 100 |

[a] $t$-values are in parentheses.

**Table 4**
$U$ is Uniform[0, 1], $V$ is Uniform[0, 2].

| Models | $1 \leq U, V \leq 10$ | | |
|---|---|---|---|
| | Model(10) | Model(11) | Model(12) |
| Constant | 0.111 (13.46) | 0.157 (19.91) | 0.193 (5.16) |
| $min\{U/V, 1\}$ | 0.333 (32.77) | 0.315 (37.99) | 0.313 (37.59) |
| $1/(U^2+V^2)^{1/2}$ | | −0.325 (−12.07) | −0.305 (−12.22) |
| $1/[V(U^2+V^2)^{1/2}]$ | −0.272 (−8.54) | | |
| $\alpha_3$ | | | 0.669 (2.94) |
| $p$-value for $H_0$: $\alpha_3 = 1$ | | | 0.146 |
| $R^2$ | 0.917 | 0.942 | 0.943 |
| Observations | 100 | 100 | 100 |

$t$ statistics in parentheses.

**Table 5**
$U$ is Uniform[0, 2], $V$ is Uniform[0, 1].

| Models | $1 \leq U, V \leq 10$ | | |
|---|---|---|---|
| | Model(10) | Model(11) | Model(12) |
| Constant | −0.0216 (−2.06) | 0.0163 (1.52) | −0.0103 (−0.91) |
| $min\{U/V, 1\}$ | 0.993 (76.42) | 0.981 (87.08) | 0.984 (89.01) |
| $1/(U^2+V^2)^{1/2}$ | | −0.266 (−7.27) | −0.419 (−4.31) |
| $1/[V(U^2+V^2)^{1/2}]$ | −0.188 (−4.62) | | |
| $\alpha_3$ | | | 1.883 (4.64) |
| $p$-value for $H_0$: $\alpha_3 = 1$ | | | 0.0296 |
| $R^2$ | 0.984 | 0.987 | 0.988 |
| Observations | 100 | 100 | 100 |

$t$ statistics in parentheses.

$\alpha_3 = 1$ shows that $\alpha_3$ is not significantly different from 1. When $F \neq G$, we get mixed result for testing $\alpha_3 = 1$, we do not reject $\alpha_3 = 1$ (at 5% level) when $V$ has a wider support than $U$, and we reject $\alpha_3 = 1$ when $V$ has a narrow support than $U$, see Tables 4 and 5 for details.

We also conduct simulations for $U, V \leq 20$ with a sample size $n = 400$. To save space we will not report detailed estimation results from $n = 400$. We only report estimated $\hat{\alpha}_3$ values and the corresponding $p$-values for testing $\alpha_3 = 1$ here. For $u, v \leq 20$ ($n = 400$), we obtain the following results: (i) for $U \sim$ Uniform[0, 1] and $V \sim$ Uniform[0, 1], we get $\hat{\alpha}_3 = 0.955$ and the $p$-value for $H_0$: $\alpha_3 = 1$ is 0.7067; (ii) for $U \sim$ Uniform[0, 1] and $V \sim$ Uniform[0, 2], we get $\hat{\alpha}_3 = 0.522$, and the $p$-value for $H_0$: $\alpha_3 = 1$ is 0.0006; (iii) for $U \sim$ Uniform[0, 2] and $V \sim$ Uniform[0, 1], we obtain $\hat{\alpha}_3 = 1.532$, and the $p$-value for $H_0$: $\alpha_3 = 1$ is 0.0002. Thus, large sample results support $\alpha_3 = 1$ when $F = G$, while it suggests that $\alpha_3 \neq 1$ when $F \neq G$.

The simulation results show that all of the proposed models fit the theoretical model very well with $R^2$ only improving slightly from model (10) to model (11), and from model (11) to model (12). However, this does not imply that one should expect that all of them should fit empirical data equally well. As we will see shortly, for the Ph.D economist job market empirical data, the goodness-of-fit $R^2$ improves significantly from model (10) to model (11), and from model (11) to model (12). Therefore, model (12) provides the best fit for the empirical data we collected.

### 2.6. A sequential matching mechanism

Up to now we have only considered a static model where all candidates and openings arrive at the market simultaneously. In this section we briefly discuss the case of a sequential matching model. Our approach follows closely that of Coles and Smith (1998) who capture a realistic feature of market search, that if a job seeker cannot match with the existing pool of vacancies, he/she will wait for the arrivals of new job vacancies. We consider two extreme cases: (i) All matched pairs can break up an earlier match and re-match in a later period without a cost. The time discount rate is zero. (ii) Both the re-match cost and the time discount rate are infinite.

**A zero re-matching cost and a zero time discount rate**

It is easy to see that in this case the results of Sections 2.2 and 2.3 remain valid without changes. Suppose at period $t$, we have a cumulative of $U_t$ job candidates, and a cumulative of $V_t$ vacancies, the number of matches will be exactly the same as in the static case with a total number of $U_t$ candidates and $V_t$ vacancies. This is because all matched pairs can freely break up with earlier matches and find the best match available to them. The highest quality individual will match with the best job available provided her quality meets the type of that job. The second highest quality individual will match with the next best available job. Consequently, the matching results will be identical as in the static case with the same total numbers of candidates and vacancies.

**An infinite re-matching cost and an infinite time discount rate**

When both the cost of re-entering the market and the time discount rate are infinite, all firms and individuals will try to find a match as soon as possible, and when a match is found, the matched pair will exit the market. Although these assumptions are not realistic, they serve as a benchmark case and from which we can deduct useful information on the more realistic finite re-matching cost/discount factor cases.

We will only consider the case where the number of candidates equals the number of openings. This will be the case if candidates and openings arrive at the market in pairs so that the total numbers of candidates and openings equal each other at all times. We assume that different pairs arrive at the market sequentially. If the first pair of candidate and opening matches with each other, they will sign a contract and exit the market. If not, they become stock and wait for matching opportunities among future arrivals. When a pair (a candidate and an opening) arrives at the market, the two can match with each other or match with the existing stock, according to whichever gives the higher utility. If no match is found, they become stock.

**The case of** $V = U = 1$.

In this case we have $\bar{P}[U \geq V] = 1/2$ as before, which gives $\bar{A}_1 = \bar{P}(1) = 1/2$.

**The case of** $V = U = 2$.

Let $u_j$ ($v_j$) be the $j$th arrival of candidates (openings), $j = 1, 2$. Because candidates and openings arrive sequentially, we cannot use the order statistics to compute $\bar{P}(r)$. However, the result obtained earlier can help calculate the matching probabilities.

The total number of different rankings of $u_1$, $u_2$, $v_1$ and $v_2$ is $4! = 24$ (four times that of the order statistic case). We can use the calculation of $\Pr(r)$ to help us to obtain $\bar{P}(r)$. For example, in the market of simultaneous arrival, the order statistic that no one finds a job is: $u_{(1)} < u_{(2)} < v_{(1)} < v_{(2)}$, and the probability is $\Pr(0) = 1/6$. In the market of sequential arrival, there are four cases that no one finds a job: (i) $u_1 < u_2 < v_1 < v_2$, (ii) $u_2 < u_1 < v_1 < v_2$, (iii) $u_1 < u_2 < v_2 < v_1$, and (iv) $u_2 < u_1 < v_2 < v_1$, giving $\bar{P}(0) = 4/24 = 1/6$. So the probability that no one finds a job remains unchanged.

There is only one case that results in different matching probabilities between a sequential market and a simultaneous market. In the case of $v_1 < u_2 < v_2 < u_1$, $u_1$ will match with $v_1$ and then $(u_1, v_1)$ exit the market. In the second period, $v_2$ and $u_2$ arrive at the market but they cannot match because $u_2 < v_2$. If the two pairs had arrived simultaneously, there would be two matched pairs, $u_1$ with $v_2$, and $u_2$ with $v_2$. So we see that when arrivals are sequential, the matching probability decreases and the market becomes less efficient. Using (A.1) in Appendix A we obtain:

$\bar{P}(0) = 4/24 = 1/6$ (=4/24 as in the simultaneous arrival case)

$\bar{P}(1) = (12 + 1)/24 = 13/24$ (it was $12/24 = 1/2$ in the simultaneous arrival case)

$\bar{P}(2) = (8-1)/24 = 7/24$ (it was $8/24 = 1/3$ in the simultaneous arrival case).

Thus, $\bar{A}_2 = (1/2)\sum_{r=0}^{2} r\bar{P}(r) = [(13/24) + 2(7/24)]/2 = 27/48 > 1/2 = \bar{A}_1$.

We still observe that as the total number of candidates and openings goes up, the average matching probability increases. However, $\bar{A}_2 = 27/48 < 28/48 = A_2$, the market of sequential arrival is less efficient compared to the case that all the candidates and openings arrive simultaneously, which is an expected result since sequential trading may lead to a very high quality candidate to match with a vacancy with a very low type, resulting in a less efficient market.

Let $\Pr[(u_i, v_j)]$ denote the probability that $u_i$ matches $v_j$. Then conditional on $u_1 < v_1$ (so that $u_1$ and $v_1$ become stock), it is easy to show that $\Pr[(u_2, v_2)] = 1/3 > \Pr[(u_2, v_1)] = 1/4$. Thus, our matching mechanism implies that $u_2$ has a lower matching probability, or higher rejection rate, when meeting with $v_1$ (an opening from the stock) than when meeting with $v_2$ (a random draw from the distribution of job openings). This is because the openings from the stock have a higher mean value than those drawn from the population. As is easily shown, our model implies that the rejection rate between a candidate from the flow and an opening from the stock increases as the size of the stock increases, or equivalently as the averaging matching probability increases (as argued by Petrongolo and Pissarides (2001, p. 406)). This is because the mean of the stock of openings goes up as its size goes up, resulting in a higher rejection rate for a given pair. Nevertheless, the average matching probability still goes up since there are more matching opportunities as the market gets thicker. It differs from the matching mechanism of Coles and Smith (1998) who assume that a firm has a constant probability to match a candidate.

Table 6 reports the simulated values of $\bar{A}_n$ for $n$ from 1 to 1000 (based on 100,000 replications). Since $\bar{A}_2 = 27/48 = 0.5625$, we see again that the simulated value matches the true value in the first three decimals.

In Table 6, we observe similar phenomena as in the case of simultaneous trading, i.e., $\bar{A}_n$ increases (with a decreasing rate) while the standard deviation of $\bar{A}_n$ decreases as $n$ increases. The dashed line in Fig. 1 shows that, as expected, the $\bar{A}_n$ curve is lower than the solid line of the $A_n$ curve. A larger friction exists in a market of sequential arrivals.

Even though sequential arrival has higher friction, the conclusion that a thick market is more efficient than a thin market remains the same as in the case of a simultaneous arrival. Further, it can be shown that $\bar{A}_n \to 1$ as $n \to \infty$.

So far we consider two extreme cases: zero time discount rate and re-match cost versus infinite time discount rate and re-match cost. Complete discussions of more realistic cases where the time discount rate and the re-match cost are some finite positive numbers are left for future research.

We do not yet consider searching cost in our model. Adding a fixed searching cost will not alter any of the conclusions obtained earlier. A variable searching cost may reduce matching probability. Given the rapid improvement of internet searching, it seems that a fixed search cost is appropriate for most situations such as economics new PhDs market. We leave more detailed discussions on extensions such as strategic trading behavior and variable searching cost to future research work.

### 2.7. Matching with reservation wage

In this section we consider the case that candidates have reservation wage. In practice it is likely that an individual with (a high) productivity $u$ only accepts offers $v$ that are not too far below $u$. Given the complexity of the problem we will investigate matching probabilities using simulations. For simplicity we only consider the case of $V = U = n$ for $n = 1, \ldots, 100$. We first sort $u$ and $v$ to get order statistics $u_1 \leq u_2 \leq \cdots \leq u_n$, and $v_1 \leq v_2 \leq \cdots \leq v_n$. We assume that an individual with productivity $u_i$ only takes offer with $v_j$ with $n - j + 1 \leq 2(n - i + 1)$, or equivalently, $j \geq 2i - n - 1$. For example, if $n = 100$, the 5th ranked individual is $u_{96}$ ($i = n - 5 + 1 = 96$) who only takes offers $v_j$ with $j \geq 2(96) - 100 - 1 = 91$. Similarly, for the 10th ranked individual (with $i = 91$) will only takes offers $v_j$ with $j \geq 81$, and that $u_i$

**Table 6**
Matching probabilities: Vacancies and candidates arrive sequentially ($n = V = U$).

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{A}_n$ | 0.5003 | 0.5622 | 0.5999 | 0.6257 | 0.6452 | 0.6603 | 0.6728 | 0.6833 | 0.6922 | 0.7001 |
| Std of $\bar{A}_n$ | 0.5001 | 0.3330 | 0.2665 | 0.2273 | 0.2034 | 0.1846 | 0.1694 | 0.1575 | 0.1475 | 0.1403 |
| $n$ | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 1000 |
| $\bar{A}_n$ | 0.7486 | 0.7736 | 0.7905 | 0.8030 | 0.8127 | 0.8207 | 0.8272 | 0.8332 | 0.8383 | 0.9248 |
| Std of $\bar{A}_n$ | 0.0977 | 0.0812 | 0.0713 | 0.0632 | 0.0582 | 0.0538 | 0.0515 | 0.0486 | 0.0461 | 0.0146 |

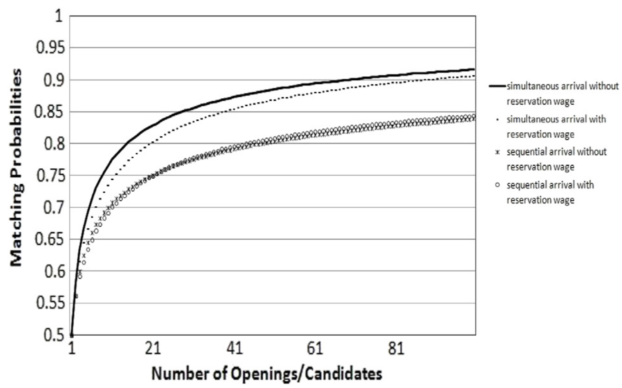


**Fig. 3.** Matching probability with/without reservation wage.

will take any offers if $i \leq 51$ (ranks 50th or below). The simulated matching probabilities are given in Fig. 3.

In Fig. 3 the top curve (solid line) corresponds to the case of simultaneous arrival without reservation wage, and the second highest curve (dotted line) is the simultaneous arrival with reservation wage. As expected we observe that with reservation wage the matching probability is slightly reduced compared with the case without reservation wage. The monotone property of the matching function remains unchanged, implying that a thick market is more efficient than a thin market whether individuals have reservation wages or not.

The next two curves correspond to the sequential arrival case with (circled line) and without reservation wage. It is interesting to observe that when vacancies and candidates arrive the market sequentially, the case with reservation wage can have a higher matching probability than without reservation wage (especially when $n$ is large). With reservation wage individuals with high productivities will refrain from taking low quality (low pay) jobs, they will wait until better openings become available at future periods, leaving the low quality jobs to low productive individuals. This leads to a more efficient matching market than the case without reservation wage (with sequential arrivals) especially in a thick market (i.e., when $n$ is large).

### *2.8. Matching quality and profits versus market size*

Up to now we have focused on examining the relationship of matching probability and market size. A referee suggested to us that one can also compute average matching quality as defined by the mean value of $uv$ for the matched pairs $(u, v)$, and examine its relationship with market size. Other quantities one can consider include average profit for firms, i.e., average value of $uv - v^2$ for firms that made a hire (a matched pair $(u, v)$), average value of $v$ for candidates who matched a job $v$ (average profit for job candidates), and average total profit, which is defined as the average profit of candidates plus the average profit of firms. We will use $E(uv)$, $E(uv - v^2)$, $E(v)$ and $E(v + uv - v^2)$ to denote the above average quantities. We draw $u$'s and $v$'s from uniform[0,1] with $n = U = V = 1, 2, \ldots, 10, 20, 30, \ldots, 100, 1000$, and the number of simulations is 100,000. The results are reported in Table 7.

From Table 7 we see that $E(uv)$ increases with the market size $n$, a thick market produces better match on average. For firm's average profit, we observe that it first rises from $n = 1$ to $n = 3$, then it decreases monotonically toward 0. It can be shown that $E(uv - v^2) \to 0$ as $n \to \infty$. This is quite intuitive because as $n \to \infty$, both $u$'s and $v$'s become dense in [0, 1], which makes all the matched pairs with $u$ very close to $v$, giving firms' average profit close to 0. Candidates average profit $E(v)$ increases from $1/6 = 0.1667$ for $n = 1$ monotonically to $1/2$ as $n \to \infty$.[3] The total average profit $E(uv - v^2 + v)$ also monotonically increases to $1/2$ as $n \to \infty$. This is as expected because firm's average profit converges to 0, and candidates' average profit converges to $1/2$, hence, the sum of the two converges to $0 + 1/2 = 1/2$ as $n \to \infty$. In our setup, when the market is very thick ($n$ large), all economic rent goes to candidates. This is because when $n$ is large, for the matched pair $(u, v)$, the candidate's productivity $u$ is expected to be sufficiently close to $v$ (from above since we need $u \geq v$ for a match), so the candidate's profit $v$ is arbitrarily close to $u$, its mean value tends to unconditional mean of $u$, which is $1/2$; while firm's profit $uv - v^2$ will be very close to zero because $u$ is very close to $v$ when $n$ is large, hence, firm's mean profit converges to 0 as $n$ gets large. We see that a thick market is more efficient than a thin market not only that it leads a higher match probability, but also that it results in a higher matching quality and a higher total average profit.

## 3. Data collection and an empirical example

Empirical study of this issue can be very difficult. It is relatively simple to collect information about successfully completed transactions in a particular market. However, gathering data about the participants who failed to complete transactions is often rather difficult.

### *3.1. The data*

The job market for new PhD economists, therefore, provides an excellent opportunity for exactly such an empirical study of thin and thick markets' performances. First, we must identify the levels of supply and demand for this market. The information we require to determine market demand is available through the journal *Job Openings for Economists* (JOE). The problem of information asymmetry is minimized when we consider the job market in economics because JOE provides virtually complete information sets for the supply of the academic jobs in the U.S. In other labor markets, we often do not know what specific information sets job applicants can access; but in this case we do because the journal is widely available to candidates going on the job market. In addition, we may determine the level of market supply by

[3] It is easy to show that $E(v) = 1/6$ for $n = 1$ by noting that $v$ can be viewed as an order statistic $v = z_1 < z_2 = u$ because $v \leq u$ for a match pair $(u, v)$. Also, $E(v) \to 1/2$ as $n \to \infty$ can be proved using similar arguments as in the proof of Lemma 1.

**Table 7**
Matching quality/profit based on 100,000 simulations ($n = V = U$).

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $E(uv)$ | 0.1244 | 0.1644 | 0.1877 | 0.2027 | 0.2142 | 0.2234 | 0.2307 | 0.2412 | 0.2457 | 0.2713 |
| $E(uv - v^2)$ | 0.0416 | 0.0473 | 0.0490 | 0.0489 | 0.0487 | 0.0481 | 0.0474 | 0.0464 | 0.0456 | 0.0448 |
| $E(v)$ | 0.1668 | 0.2166 | 0.2477 | 0.2703 | 0.2862 | 0.2992 | 0.3098 | 0.3192 | 0.3268 | 0.3434 |
| $E(uv - v^2 + v)$ | 0.2074 | 0.2639 | 0.2968 | 0.3192 | 0.3350 | 0.3473 | 0.3572 | 0.3656 | 0.3724 | 0.3881 |
| $n$ | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 1000 |
| $E(uv)$ | 0.2713 | 0.2821 | 0.2886 | 0.2936 | 0.2968 | 0.2997 | 0.3018 | 0.3036 | 0.3049 | 0.3246 |
| $E(uv - v^2)$ | 0.0377 | 0.0341 | 0.0312 | 0.0290 | 0.0271 | 0.0257 | 0.0245 | 0.0232 | 0.0224 | 0.0082 |
| $E(v)$ | 0.3760 | 0.3947 | 0.4068 | 0.4155 | 0.4223 | 0.4272 | 0.4317 | 0.4353 | 0.4384 | 0.4794 |
| $E(uv - v^2 + v)$ | 0.4137 | 0.4288 | 0.4380 | 0.4445 | 0.4495 | 0.4529 | 0.4562 | 0.4585 | 0.4608 | 0.4876 |

**Table 8**
Summary of academic markets for new PhD economists.

| Fields with most openings | Average openings | # of filled positions | Probability of matching | Number of candidates |
|---|---|---|---|---|
| Year 2000 | | | | |
| Any field(AF) | 95.3 | 42 | 0.441 | 0 |
| Macro(E0) | 49.3 | 30 | 0.608 | 83 |
| Micro(D0) | 36.2 | 18 | 0.498 | 42 |
| International(F0) | 34.9 | 25 | 0.717 | 39 |
| Econometrics(C1) | 33.5 | 13 | 0.388 | 43 |
| Financial Econ(G0) | 33.4 | 19 | 0.568 | 39 |
| Agric Econ(Q0) | 25.6 | 13 | 0.507 | 16 |
| Public Econ(H0) | 25.4 | 9 | 0.354 | 37 |
| General Econ(A1) | 22.6 | 4 | 0.177 | 0 |
| Health Econ(I1) | 21.2 | 9 | 0.426 | 11 |
| IO(L0) | 19.6 | 15 | 0.765 | 60 |
| Mean of remaining fields | 2.96 | 1.40 | 0.305 | 2.16 |
| Total | 617 | 308 | 0.499 | 529 |
| Year 2001 | | | | |
| Any field(AF) | 125.0 | 64 | 0.512 | 0 |
| Macro(E0) | 54.8 | 31 | 0.566 | 72 |
| International(F0) | 39.6 | 11 | 0.277 | 29 |
| Micro(D0) | 38.2 | 20 | 0.523 | 34 |
| Agric Econ(Q0) | 37.9 | 13 | 0.343 | 11 |
| Econometrics(C1) | 36.0 | 13 | 0.361 | 32 |
| Health Econ(I1) | 34.5 | 18 | 0.521 | 14 |
| Financial Econ(G0) | 31.9 | 16 | 0.501 | 24 |
| Public Econ(H0) | 20.7 | 11 | 0.532 | 25 |
| IO(L0) | 20.7 | 15 | 0.726 | 59 |
| General Econ(A1) | 18.3 | 3 | 0.164 | 0 |
| Mean of remaining fields | 3.27 | 1.20 | 0.268 | 1.92 |
| Total | 696 | 308 | 0.443 | 445 |

contacting graduate programs in economics regarding their PhDs who have gone on the job market in the past several years.

Our data is organized by field. The definition of the field can be found in the "Classification System of Journal Articles" by the *Journal of Economic Literature*. In particular, we use the field consisting of a capital letter and a numeral. For example, E0 means "Macroeconomics and Monetary Economics".

We collect the American academic job openings listed in the September, October, November, and December issues of JOE in 1999 and 2000. In addition, we collect the information for job candidates and the information on whether each opening if filled in the fall. The detailed descriptions for data collection and construction is listed in the Appendix.

The summary information of the markets is listed in Table 8. In addition to showing the ten fields with the most job openings in the table, we include any field (AF), the mean of the remaining fields not listed in the table, and the whole market. In both years, AF is by far the largest "field". Macroeconomics (E0), Microeconomics (D0), and International Economics (F0) were the top three fields other than AF in both years. In 1999, the mean of the matching probabilities in the ten fields with the most job openings is 0.501, while the mean of the matching probabilities in the remaining fields is 0.305. Thicker fields do have larger matching probabilities than thinner fields. The same pattern repeats in 2000 where the mean of the matching probabilities for the ten fields with the largest demand is 0.451, while the rest of the fields have an average matching probability of 0.268.

### 3.2. Parametric estimation results

We estimate our proposed matching models, given in (10)–(12), using the collected data. We are primarily interested in the sign of the coefficient for the variable of thickness, measured by the variable $d = (\text{candidates}^2 + \text{openings}^2)^{1/2}$.

Table 9 gives the estimation results of models (10)–(12), in the same format as that of Table 3. It is clear that the regressions based on model (12) have the best fit, followed by model (11). In all these different specifications using different sample data, the parameter estimates of $\alpha_2$, the coefficient of the inverse of the thickness variable $d$ in models (10)–(12), are negative. Moreover, they are significant at the 5% level for eight out of nine cases,[4] and are all significant at the 10% level (note that it is an one-sided test). Thus our estimation results predict that the matching

[4] Models (10), (11), (12), each has three cases correspond to using 2000, 2001 and pooled data, respectively.

**Table 9**
Regressions of matching probabilities. (U.S. academic market for new PhD Economists.)

| | Model (10) | Model (11) | Model (12) |
|---|---|---|---|
| Job market in January 2000 | | | |
| Constant | 0.284 | 0.362 | 0.847 |
| | (5.69)[a] | (6.67) | (1.98) |
| $\min\{U/V, 1\}$ | 0.208 | 0.142 | 0.020 |
| | (2.51) | (1.75) | (.237) |
| $1/[(V^2+U^2)^{1/2}]$ | | −0.112 | −0.674 |
| | | (−4.03) | (−1.52) |
| $1/[V(V^2+U^2)^{1/2}]$ | −0.017 | | |
| | (−2.72) | | |
| $\alpha_3$ | | | 0.223 |
| | | | (1.30) |
| $R^2$ | 0.250 | 0.339 | 0.466 |
| # of observations | 61 | 61 | 61 |
| Job market in January 2001 | | | |
| Constant | 0.370 | 0.456 | 0.658 |
| | (8.29) | (8.97) | (3.36) |
| $\min\{U/V, 1\}$ | −0.023 | −0.094 | −0.155 |
| | (−0.29) | (−1.19) | (−1.84) |
| $1/[(V^2+U^2)^{1/2}]$ | | −0.212 | −0.468 |
| | | (−4.78) | (−2.20) |
| $1/[V(V^2+U^2)^{1/2}]$ | −0.061 | | |
| | (−3.80) | | |
| $\alpha_3$ | | | 0.411 |
| | | | (1.70) |
| $R^2$ | 0.192 | 0.273 | 0.315 |
| # of observations | 65 | 65 | 65 |
| Pooled sample of 2000 and 2001 | | | |
| Constant | 0.314 | 0.398 | 0.775 |
| | (9.43) | (10.88) | (3.36) |
| $\min\{U/V, 1\}$ | 0.106 | 0.035 | −0.068 |
| | (1.82) | (0.62) | (−1.15) |
| $1/[(V^2+U^2)^{1/2}]$ | | −0.139 | −0.591 |
| | | (−5.97) | (−2.45) |
| $1/[V(V^2+U^2)^{1/2}]$ | −0.023 | | |
| | (−3.94) | | |
| $\alpha_3$ | | | 0.274 |
| | | | (2.03) |
| $R^2$ | 0.160 | 0.266 | 0.369 |
| # of observations | 126 | 126 | 126 |

[a] $t$-values are in parentheses.

probability increases as the market becomes thicker, consistent with the main prediction of our theoretical model. In other words, a thicker market produces a higher probability of matching. The statistically insignificant estimates of $\alpha_1$ reflect the fact that the number of candidates in top-50 schools in each field is a noisy measure. One source of noise comes from the classification of candidates fields. For example, students often indicate their fields to be one of the thicker fields. In the two year period we have data, among the 126 fields that have academic openings, only 45% of them have candidates, although 62% of those fields have some success to hire at least one candidate.

Fig. 4 gives the estimated curves using 2000 job market data, Fig. 5 uses 2001 job market data, and Fig. 6 uses the pooled sample. These figures graph the observed and predicted matching probabilities for models (11) and (12). Each point in these figures represents one field. The dotted line in each figure represents the predicted probability based on model (11); the solid line plots the predicted probability based on model (12). The prediction is carried at the sample mean of $\min\{U/V, 1\}$. Comparing model (11) with model (12) we observe that the nonlinear model shows more pronounced thickness effects. Within the same model (say model (12)), all three graphs are similar, reflecting the fact that estimates from different sample are similar. From all the graphs, we clearly see that matching probability is an increasing and concave function of the thickness ($d$) of the market.

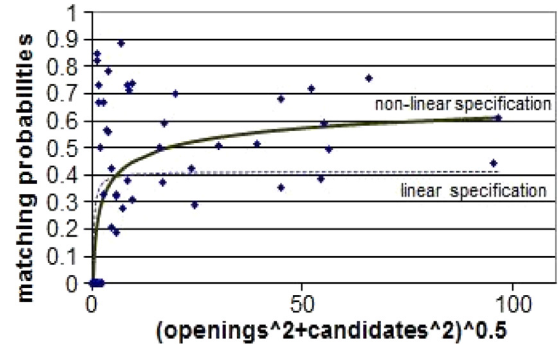


**Fig. 4.** January 2000 job market.

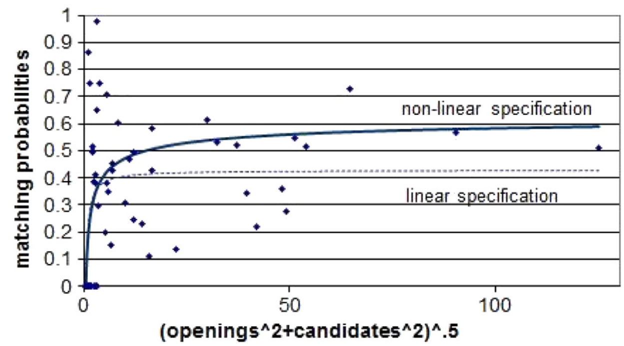


**Fig. 5.** January 2001 job market.

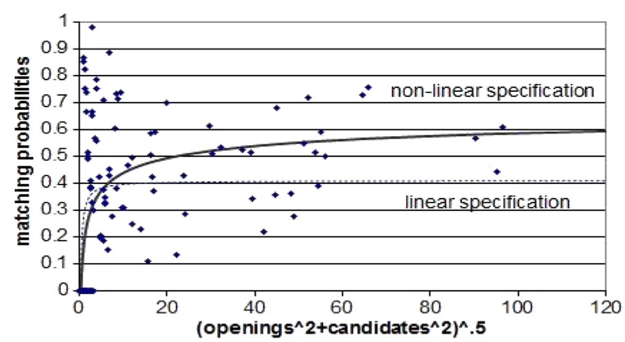


**Fig. 6.** January 2000–2001 job market.

To understand the magnitude of the effect of thickness on the matching probability, consider model (12), where the number of candidates equals the number of job openings; we have that (when $U = V$) the matching probability is

$$\widehat{(M/V)} = \hat{\alpha}_0 + \hat{\alpha}_1(1) + \hat{\alpha}_2/d^{\hat{\alpha}_3},$$

where $d = \sqrt{2}V$ (since $U = V$). For $U = V = 5$, 10, and 50, and using the pooled sample estimation result, our empirical model predicts the matching probabilities of 0.361, 0.421, and 0.523, respectively.

Table 10 reports market size, $d = \sqrt{U^2+V^2}$, frequency of candidates finding academic jobs, $M/V$, where $M$ is the number of positions filled, estimated probabilities of finding academic jobs for the six popular fields: Macro, Micro, Econometrics, International, Public, Health using data of 2000 and 2001. We see that for most cases the estimated matching probabilities are close to the observed matching frequencies except for the field 'International' which has a very high matching rate (71.7%) in 2000 but has a quite low matching rate (27.8%) in 2001 although the market thickness are similar for these two year ($d = 52.3$ and $d = 49.1$, respectively). Looking into the data in more detail we found that

**Table 10**
Matching probabilities with selected fields.

| | $d$ | 2000 frequency | Estimated | $d$ | 2001 frequency | Estimated |
|---|---|---|---|---|---|---|
| Econometrics | 54.51 | 0.388 | 0.509 | 48.19 | 0.361 | 0.510 |
| Micro | 56.19 | 0.498 | 0.511 | 51.30 | 0.547 | 0.514 |
| Macro | 96.55 | 0.608 | 0.538 | 90.48 | 0.566 | 0.535 |
| International | 52.33 | 0.717 | 0.507 | 49.10 | 0.278 | 0.522 |
| Public | 44.86 | 0.355 | 0.499 | 32.43 | 0.532 | 0.479 |
| Health | 23.81 | 0.426 | 0.492 | 37.26 | 0.521 | 0.528 |

for 'international', $U = 25$ in 2000, but it drops more than 50% to $U = 11$ in 2001. The substantial supply shock explains the huge drop in the observed matching (frequency) probability in 2001.

In order to check whether fields such as "any field" (AF) and "general economics" (A1) contaminate our estimation results, we conduct estimates removing fields "AF" and "A1". These two fields have large numbers of openings while there are no candidates labeled as "any field" or "general economics". Estimation results not reported here (they are available upon request) show virtually identical parameters estimates as well as the goodness-of-fit $R^2$ to the results given in Table 9. Thus the fact that there are zero candidates in the thick fields "AF" and "A1" does not affect our estimation results nor the conclusions derived from them.

### 3.3. Nonparametric estimation results

Finally, we use a nonparametric method to estimate the matching probability function as a function of market thickness $d$ ($d = \sqrt{U^2 + V^2}$). Although the most popular nonparametric estimation method is the kernel method, kernel method (with a fixed bandwidth) is not appropriate for estimating the matching function with the data we have. Because our data is noise and are highly unevenly distributed. Using a constant bandwidth is not appropriate for this kind of data because there is not enough data at the tail part of the distribution (large value of $d$). The nonparametric k-nearest-neighbor (knn) method is more suitable to our data case because the knn method always uses $k$ data points to estimate the matching function at any given point. Therefore, we will use the knn method to estimate the matching function nonparametrically. Since nonparametric methods require large data set, we will apply knn method only to the pooled data (with sample size $n = 126$). We select $k$ using the leave-one-out cross validation method. The estimation result is given in Fig. 7.

The dashed line gives the unconstrained knn estimate for the matching function. First we observe that nonparametric estimated curve has a similar shape as that of the parametric method estimated curve. Secondly, we see that for the most part the curve increases monotonically as $n$ grows. There are some bumps around $n = 5$ and $n = 10$. We also estimate a constrained matching function imposing the monotone restriction, e.g., Hall and Huang (2001), Li et al. (2015a,b).[5] The estimated result is plotted as the dotted line in Fig. 7. The constrained estimation method removes the bumps and results in a monotone curve. The goodness-of-fit $R^2$ for the nonparametric knn estimated model with and without monotone restriction is $R^2_{knn,m} = 0.417$ $R^2_{knn} = 0.444$, respectively. It is not surprising that nonparametric estimation methods result in higher $R^2$ than parametric models as nonparametric fitted curves are less smooth than parametric models, and therefore can trace the in-sample data better than parametric models.

To formally test whether the parametric model (12) adequately describes the data, we use the nonparametric specification test

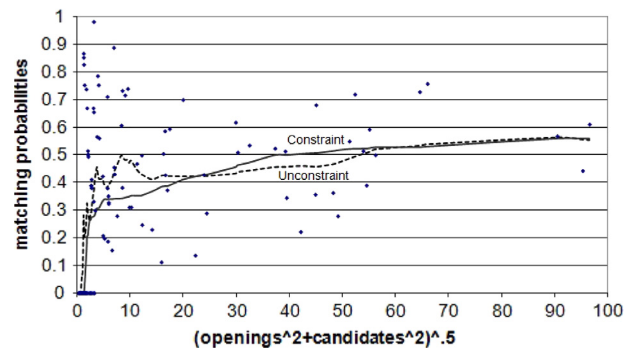


**Fig. 7.** Nonparametric Knn estimation.

proposed by Li et al. (2015a,b), who generalize Zheng (1996), and Li and Wang's (1998) kernel based test to knn based testing framework, to test the null hypothesis that the model (12) fitted curve and the nonparametric knn method fitted curve do not differ from each other significantly. Following Li et al. (2015a,b) we use the wild bootstrap method to generate 1000 bootstrap statistics, from which we compute the $p$-value for the test statistic. We obtain a $p$-value of 0.36. Therefore, we failed to reject the null hypothesis at any convention level, and we conclude that the parametric and the nonparametric methods give similar estimated shapes for the matching probability functions. We view this as an additional evidence supporting our finding that a thick market is more efficient than a thin market.

Before closing this section we would like to emphasize that the main contribution of our paper is to derive a matching model with microfoundation. The empirical application of economics Ph.D matching market serves as an illustrative example showing that, using the parametric matching functional form as given in (12), the empirical results are consistent with our theoretical model prediction. There are many directions one can improve the empirical work such as allowing $U$ and $V$ to be endogenously determined, see, e.g., Mortensen and Pissarides (1994), and Borowczyk-Martins et al. (2013), but this requires new/rich data set and we leave it to a future research topic.

## 4. Conclusions

In this paper we propose a matching model with the matching probability depending on the thickness of a market. In our model, the types of firms and the productivities of job candidates are randomly drawn from a common distribution. A firm employs a job applicant only if the job applicant's productivity is higher than its type.

All firms prefer a higher productivity applicant to a lower one, and all applicants prefer a higher minimum standard firm to a lower one. In this hypothetical market, we show that the probabilities of matches in a thin market differ significantly from those in a thick market.

We also characterize the case where firms and candidates have different distributions, the case where the number of openings does not equal the number of applicants, the case where openings

[5] Li et al. (2015a,b) generalize Hall and Huang's (2001) kernel constrained estimation method to the knn method. They apply their proposed monotonicity test to the same data as we used here to estimate a univariate nonparametric regression model by the knn method with $d = \sqrt{U^2 + V^2}$ as the regressor.

and candidates arrive at the market sequentially, and the case when candidates have reservation wages so that they will not accept offers below their reservation wages. In all these cases, the matching probability still increases with the thickness of the market. In addition, we propose a parsimonious matching function which is fairly close to the (simulated) theoretical matching function.

The matching described in this paper is assortative. We derive a theoretical matching function with micro-foundation under no friction and under perfect information in this market. The matching function exhibits strong properties that are increasing and concave in market size. Of course, any real world market with friction and imperfect information, the matching probabilities will for sure be lower. However, we believe that a matching function is increasing and concave in market size should remain valid under general settings.

To illustrate our model, we apply our matching model to the U.S. academic market for junior PhD economists. Consistent with the prediction of our model, a field with more job openings and more candidates has a higher probability of matching. In particular, according to our model, the matching probability increases from 0.361 for 5 candidates and 5 openings to 0.523 for 50 candidates and 50 openings.

One implication of our model is a large occupation or a bigger city may have a lower unemployment. Gan and Zhang (2006) provided evidence of the latter. Another potential application of the model is the marriage market. In Costa and Kahn (2000), college graduated power couples are more likely located in larger cities because of easier match for both, a likely consequence of thicker job markets in cities. It will be interesting to study how probabilities of marriage vary by the market size (by certain age, education level, etc.) of the eligible pool of both men and women.

The model above can be extended in many directions, such as to the regular labor or housing markets. It may require more serious effort to relax some assumptions made in this paper. For example, job candidates may have different preference orderings of employers, as may employers. More general transferable utility functions may be more appropriate for a general labor market because one may enjoy a higher utility if he/she moves to a better quality firm. Buyers in the housing market have several aspects to consider, while sellers may only care about the selling prices.

## Appendix

### A.1. A useful lemma

**Lemma A.1.** *Let $u_{(1)} < u_{(2)} < \cdots < u_{(U)}$ be the order statistic obtained from i.i.d. data $u_1, \ldots, u_U$, and $v_{(1)} < v_{(2)} < \cdots < v_{(V)}$ be the order statistic obtained from i.i.d data $v_1, \ldots, v_V$. $u_i$ and $v_j$ have the common distribution $F(\cdot)$ with pdf $f(\cdot)$. Let $z_{(1)} < z_{(2)} < \cdots < z_{(U+V)}$ denote the order statistic obtained from $(u_{(1)}, \ldots, u_{(U)}, v_{(1)}, \ldots, v_{(V)})$. There are $(U+V)!/[(V!)(U!)]$ such orderings. Let $\mathcal{Z}_n$ denote the random variable $(z_{(1)}, \ldots, z_{(U+V)})$. Then for any one particular order $z$, we have*

$$\Pr(\mathcal{Z}_n = z) = \frac{(V!)(U!)}{(U+V)!}. \tag{A.1}$$

In Lemma A.1, the probability of any particular order statistic $z$ does not depend on the underlying distribution of candidates and openings. Since the overall matching probability involves accounting the number of appropriate orderings, it does not depend on underlying distributions. We would like to emphasize that the proof of Lemma A.1 depends on the assumption that $u$'s and $v$'s are i.i.d draws for candidates' productivities and firms' types. There is also a possibility that there is heterogeneity and that the data in either dimension may not be independent and identically distributed. Without i.i.d assumption, Lemma A.1 may not holds.

**Proof of Lemma A.1.** The joint distribution of $(u_{(1)}, \ldots, u_{(U)})$ and $(v_{(1)}, \ldots, v_{(V)})$ is $(U!)\prod_{r=1}^{U} f(u_{(r)})$ and $(V!)\prod_{r=1}^{V} f(v_{(r)})$, respectively. Therefore, the joint distribution function for $(u_{(1)}, \ldots, u_{(U)}, v_{(1)}, \ldots, v_{(V)})$ is $(V!)(U!)[\prod_{r=1}^{V} f(u_{(r)})][\prod_{r=1}^{U} f(v_{(r)})]$.

$$\begin{aligned}
\Pr(\mathcal{Z}_n = z) &= P(z_{(1)} < z_{(2)} < \cdots < z_{(U+V)}) \\
&= (V!)(U!)\int_{\{z_{(1)}<z_{(2)}<\cdots<z_{(U+V)}\}} \prod_{r=1}^{U+V} dF(z_{(r)}) \\
&= (V!)(U!)\int_{\{z_{(2)}<z_{(3)}<\cdots<z_{(U+V)}\}} F(z_{(2)}) \prod_{r=2}^{U+V} dF(z_{(r)}) \\
&= (V!)(U!)\int_{\{z_{(3)}<\cdots<z_{(U+V)}\}} (1/2)F^2(z_{(3)}) \prod_{r=3}^{U+V} dF(z_{(r)}) \\
&= (V!)(U!)\int_{\{z_{(3)}<\cdots<z_{(U+V)}\}} (1/2)\ldots(1/(U+V-1)) \\
&\quad \times \int_a^b F^{U+V-1}(z_{(U+V)})F(z_{(U+V)}) \\
&= \frac{(V!)(U!)}{(U+V)!}.
\end{aligned}$$

### A.2. Proof of Lemma 1

We know that $A_n$ is independent of the distribution $f$. Therefore, without loss of generality we assume that $f$ is a uniform distribution in the unit interval. For any (small) $\eta > 0$, we choose $m = [1/\eta] > 1/\eta$. ([.] denotes the integer part of .) and divide the unit interval into $m$ intervals with equal length $1/m$ for each. That is: $[0, 1] = \cup_{l=1}^{m} I_l$, where $I_l = [(l-1)/m, l/m)$ $(l = 1, \ldots, m$, with $I_m = [(m-1)/m, 1])$. Let $n_{u,l}$ and $n_{v,l}$ denote the number of observations from $\{u_i\}_{i=1}^{n}$ and $\{v_i\}_{i=1}^{n}$ that fall inside in interval $I_l$ $(l = 1, \ldots, m)$. We know that on the average there are $n/m$ observations from both $\{u_i\}_{i=1}^{n}$ and $\{v_i\}_{i=1}^{n}$ that fall inside interval $I_l$ for all $l = 1, \ldots, m$. In fact by the strong law of large number (Billingsley, 1986, p. 80) we have $P(\lim_{n\to\infty} n_{s,l}/n = 1/m) = 1$ for all $l = 1, \ldots, m$ $(s = u, v)$.

Note that the candidates with $u_i$'s fall inside the interval $I_l$ can match with any job opening with $v_j$'s falls in $I_{l-1}$ $(l = 2, \ldots, m)$. Given that with probability one that $n_{u,l}/n \to 1/m$ and $n_{v,l-1}/n \to 1/m$, we know that, with probability approaching to one as $n \to \infty$, that there can have $n/m$ matches for $u_i's \in I_l$ matching with $v_i's \in I_{l-1}$. Sum over $l$ from 2 to $m$ we get, with probability one, that the number of matched candidates is at least (since we ignore the possibility that $u_i's \in I_1$ may also find match) $n_{match}/n \geq (m-1)/m \geq 1-\eta$, or more formally, we have, as $n \to \infty$,

$$P\left(\frac{n_{match}}{n} \geq 1-\eta\right) \to 1. \tag{A.2}$$

Therefore we have

$$1 \geq A_n = \frac{1}{n}\sum_{r=0}^{n} rP(r) \geq \frac{1}{n}\sum_{r\geq n(1-\eta)} rP(r) \to 1,$$

because for any $1 > \epsilon > 0$, we can choose $\eta = \epsilon/2$ and by (A.2), we have $n^{-1}\sum_{r\geq n(1-\eta)} rP(r) \geq n^{-1}n(1-\eta)\sum_{r>n(1-\eta)} P(r) \geq (1-\eta)^2 \geq 1-\epsilon$. Thus, $n^{-1}\sum_{r\geq n(1-\eta)} rP(r) \to 1$ as $n \to \infty$ which implies $A_n \to 1$, completing the proof of Lemma 1.

Let $\Pr(\# \geq r)$ denote the probability that at least $r$ people find jobs. Then it is easy to see that $\Pr(\# \geq r) = \Pr(u_{(n)} > v_{(n-r+1)}, u_{(n-1)} > v_{(n-r)}, \ldots, u_{(n-r)} > v_{(2)}, u_{(n-r+1)} > v_{(1)})$. The next lemma shows that $\Pr(\# \geq r)$ can be used to compute $E(r)$.

**Lemma 2.** *Let # denote the number of people who find jobs ($0 \leq \# \leq \min\{V, U\}$), and denote by $\Pr(\# \geq r) = \sum_{m=r}^{n} \Pr(m)$ the probability that at least $r$ people find jobs. Then*

$$E(r) = \sum_{r=1}^{n} Pr(\# \geq r).$$

**Proof.**

$$\begin{aligned} E(r) &= \sum_{r=1}^{n} r \Pr(r) = \{\Pr(1) + (2)\Pr(2) + \cdots + (n)\Pr(n)\} \\ &= \{[\Pr(1) + \Pr(2) + \cdots + \Pr(n)] + [\Pr(2) + \cdots + \Pr(n)] \\ &\quad + [\Pr(n-1) + \Pr(n)] + \Pr(n)\} \\ &= \sum_{r=1}^{n} \Pr(\# \geq r). \end{aligned}$$

### A.3. The case of $n = 3$

Let $u_3 > u_2 > u_1$ be the order statistic of candidates, and $v_3 > v_2 > v_1$ be the order statistic of openings (we omit the parentheses in the subscripts to simplify the notation).

$$\begin{aligned} \Pr(0) &= \Pr(u_3 < u_2 < u_1 < v_1 < v_2 < v_3) \\ &= (3!)^2(1/6!) = 1/20. \end{aligned}$$

$$\Pr(\# \geq 1) = 1 - \Pr(0) = 19/20.$$

$$\begin{aligned} \Pr(\# \geq 2) &= \Pr(u_3 > v_2, u_2 > v_1) \\ &= \Pr(u_3 > v_2 > u_2 > v_1) + \Pr(u_3 > u_2 > v_2 > v_1) \\ &= 14/20 \end{aligned}$$

since

$$\begin{aligned} \Pr(u_3 > v_2 > u_2 > v_1) &= \Pr(u_3 > v_3 > v_2 > u_2 > v_1 > u_1) \\ &\quad + \Pr(v_3 > u_3 > v_2 > u_2 > v_1 > u_1) \\ &\quad + \Pr(u_3 > v_3 > v_2 > u_2 > u_1 > v_1) \\ &\quad + \Pr(v_3 > u_3 > v_2 > u_2 > u_1 > v_1) \\ &= 4[(3!)/(6!)] = 4/20. \end{aligned}$$

$$\begin{aligned} \Pr(u_3 > u_2 > v_2 > v_1) &= \Pr(u_3 > u_2 > v_3 > v_2 > v_1 > u_1) \\ &\quad + \Pr(u_3 > v_3 > u_2 > v_2 > v_1 > u_1) \\ &\quad + \Pr(v_3 > u_3 > u_2 > v_2 > v_1 > u_1) \\ &\quad + \Pr(u_3 > u_2 > v_3 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > v_3 > u_2 > v_2 > u_1 > v_1) \\ &\quad + \Pr(v_3 > u_3 > u_2 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > u_2 > v_3 > u_1 > v_2 > v_1) \\ &\quad + \Pr(u_3 > v_3 > u_2 > u_1 > v_2 > v_1) \\ &\quad + \Pr(v_3 > u_3 > u_2 > u_1 > v_2 > v_1) \\ &\quad + \Pr(u_3 > u_2 > u_1 > v_3 > v_2 > v_1) \\ &= 10[(3!)/(6!)] = 10/20. \end{aligned}$$

$$\begin{aligned} \Pr(3) &= \Pr(u_3 > v_3, u_2 > v_2, u_1 > v_1) \\ &= \Pr(u_3 > v_3 > u_2 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > u_2 > v_3 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > v_3 > u_2 > u_1 > v_2 > v_1) \\ &\quad + \Pr(u_3 > u_2 > v_3 > u_1 > v_2 > v_1) \\ &\quad + \Pr(u_3 > u_2 > u_1 > v_3 > v_2 > v_1) \\ &= 5\{(3!)^2/(1/6!)\} = 5/20. \end{aligned}$$

Therefore, by Lemma 2 we have

$$\begin{aligned} A_3 &= \frac{1}{3}\sum_{r=1}^{3} \Pr(\# \geq r) \\ &= [(19/20) + (14/20) + (5/20)]/3 = 19/30. \end{aligned}$$

### A.4. The case of $n = 4$

$\Pr(0) = (4!)^2/(8!) = 1/70$ by Lemma 1. $\Pr(\# \geq 1) = 1 - P(0) = 69/70$.

$\Pr(\# \geq 2) = [(3+6)+(3+6+10)+(3+6+10+15)]/70 = 62/70$.

$\Pr(\# \geq 3) = [(2+3)+(2+3+4)+(2+3+4+5)]/70 = 42/70$.

$\Pr(4) = [(2+3)+(2+3+4)]/70 = 14/70$. Therefore, by Lemma 3 we have

$$\begin{aligned} A_4 &= \frac{1}{4}\sum_{r=1}^{4} \Pr(\# \geq r) \\ &= [(69/70) + (62/70) + (42/70) + (14/70)]/4 = 187/280. \end{aligned}$$

### A.5. The case two uniform distribution with different means

We assume that sellers are randomly drawn from a uniform distribution in the unit interval (unif[0, 1]), while the buyers are random draws with a uniform distribution in the interval of $[\delta, 1+\delta]$. We only consider the case of $V = U = n$.

For $n = 1$, straightforward calculation shows that $A_1 = \Pr(1) = (1/2)(1-\delta)^2$.

For $n = 2$, a more tedious calculation shows that

$$Pr(0) = (1 + 4\delta + 6\delta^2 - 4\delta^3 - \delta^4)/6, \quad \text{and}$$

$$\Pr(2) = (1-\delta)^4/3.$$

Hence (using $Pr(1) = 1 - Pr(0) - Pr(2)$),

$$\begin{aligned} A_2 &= (1/2)[\Pr(1) + 2\Pr(2)] \\ &= (7/12)(1-\delta)^2 + (1/12)\delta(1-\delta)^2(2+3\delta). \end{aligned}$$

### A.6. Computing $a_2$ for general $f$ and $g$

For $n = 2$, let $u_1 < u_2$ be order statistics drawn from $F$, and $v_1 < v_2$ from $G$. The marginal PDFs and CDFs for $u_1$ and $u_2$ are, $f_1(u) = 2[1-F(u)]f(u), F_1(u) = 2F(u)-F^2(u), f_2(u) = 2F(u)f(u), F_2(u) = F^2(u)$ Similarly, the marginal PDFs and CDFs for $v_1$ and $v_2$ are, $g_1(u) = 2[1-G(u)]g(u)$, $G_1(u) = 2G(u) - G^2(u)$, $g_2(u) = 2G(u)g(u)$ and $G_2(u) = G^2(u)$, where $f$ and $g$ are underlying density functions for $u$ and $v$. From $\sum_{r=1}^{n} r\Pr(r) = \sum_{i=1}^{n} Pr(r \geq i)$ we know that for computing $A_2$, we only need to calculate $\Pr(r \geq 1)$ and $\Pr(r = 2)$.

$$\begin{aligned} \Pr(r = 0) &= \Pr(u_2 < v_1) \\ &= \int\int_{u_2}^{\infty} dG_1(v_1)dF_2(u_2) \\ &= 1 - 2\int [2G(u) - G^2(u)]F(u)dF(u), \end{aligned}$$

where the second equality holds by substituting the expressions of $G_1(\cdot)$ and $F_2(\cdot)$. Therefore, we have,

$$\begin{aligned} \Pr(r \geq 1) &= 1 - \Pr(r = 0) \\ &= 2\int [2G(u) - G^2(u)]F(u)dF(u). \end{aligned} \tag{A.3}$$

To calculate $\Pr(r = 2)$, we consider two cases. (i) $v_2 < u_1$ and (ii) $v_1 < u_1 < v_2 < u_2$. For case (i)

$$\begin{aligned} \Pr(v_2 < u_1) &= \int\int_{-\infty}^{u_1} dG_2(v_2)dF_1(u_1) \\ &= 2\int G^2(u)[1 - F(u)]dF(u), \end{aligned} \tag{A.4}$$

where the second equality follows from $G_2(u) = G^2(u)$ and $dF_1(u) = 2[1 - F(u)]dF(u)$.

For case (ii) by noting that the joint PDF of $(u_1, u_2, v_1, v_2)$ is $4f(u_1)f(u_2)g(v_1)g(v_2)$, we have,

$$\begin{aligned}
&\Pr(v_1 < u_1 < v_2 < u_2) \\
&= \int_{\{v_1<u_1<v_2<u_2\}} 4f(u_1)f(u_2)g(v_1)g(v_2)d(u_1,u_2,v_1,v_2) \\
&= 4\int_{\{u_1<v_2<u_2\}} \left[\int_{-\infty}^{u_1} g(v_1)dv_1\right] f(u_1)f(u_2)g(v_2)d(u_1,u_2,v_2) \\
&= 4\int_{\{u_1<v_2<u_2\}} G(u_1)f(u_1)f(u_2)g(v_2)d(u_1,u_2,v_2) \\
&= 4\int_{\{u_1<u_2\}} \left[\int_{u_1}^{u_2} g(v_2)dv_2\right] G(u_1)f(u_1)f(u_2)d(u_1,u_2) \\
&= 4\int_{\{u_1<u_2\}} [G(u_2)-G(u_1)]\,G(u_1)f(u_1)f(u_2)d(u_1,u_2) \\
&= 4\int \left[G(u_2)\int_{-\infty}^{u_2} G(u_1)dF(u_1) - \int_{-\infty}^{u_2} G^2(u_1)dF(u_1)\right] dF(u_2) \\
&= 4\int \left[G(u)\int_{-\infty}^{u} G(s)dF(s) - \int_{-\infty}^{u} G^2(s)dF(s)\right] dF(u). \qquad (A.5)
\end{aligned}$$

Gathering results in (A.3)–(A.5), we have

$$\begin{aligned}
A_2 &= \frac{E(r)}{2} = \frac{1}{2}\sum_{i=1}^{2} \Pr(r \geq i) \\
&= \frac{1}{2}[Pr(r \geq 1) + \Pr(v_2 < u_1) + \Pr(v_1 < u_1 < v_2 < u_2)] \\
&= \int \Big\{2G(u)F(u) - 2G^2(u)F(u) + G^2(u) \\
&\quad + 2\left[G(u)\int_{-\infty}^{u} G(s)dF(s) - \int_{-\infty}^{u} G^2(s)dF(s)\right]\Big\} dF(u).
\end{aligned}$$

### A.7. The case of $v \neq u$

**Case (i)** $(U, V) = (1, 2)$ or $(2, 1)$

Let $v_1 < v_2$ be the order statistic of openings. By Lemma 1 we have

$$\begin{aligned}
\Pr(1) &= \Pr(v_2 > u > v_1 \text{ or } u > v_1, v_2) \\
&= \Pr(v_2 > u > v_1) + \Pr(u > v_2 > u_1) = 2\{1!2!/3!\} = 2/3.
\end{aligned}$$

Therefore, $M_{U,V} = M_{1,2} = \Pr(1) = 2/3$. From this one can compute $B_{U,V}$.

**Case (ii)** $(U, V) = (3, 1)$ or $(1, 3)$

Let $v_1 < v_2 < v_3$ be the order statistic of openings.

$\Pr(0) = \Pr(u < v_1, v_2, v_3) = \Pr(u < v_1 < v_2 < v_3) = \{1!3!/4!\} = 1/4$.

$\Pr(1) = 1 - \Pr(0) = 3/4$. Therefore, $M_{1,3} = \Pr(1) = 3/4$. Then one can compute $B_{U,V}$.

**Case (iii)** $(U, V) = (2, 4)$ or $(4, 2)$

Let $u_1 < u_2$ and $v_1 < v_2 < v_3 < v_4$ be the order statistics of candidates and openings, respectively.

$\Pr(0) = \Pr(u_2 < v_2) = Pr(u_1 < u_2 < v_1 < v_2 < v_3 < v_4) = \{2!4!/6!\} = 1/15$.

$\Pr(1) = \Pr(v_1 < u_1 < u_2 < v_2) + \Pr(v_1 < u_1 < v_2) = 1/15 + 4/15 = 1/3$.

$\Pr(2) = 1 - \Pr(0) - \Pr(1) = 3/5$. Hence, $M_{2,4} = 1/3 + 2(3/5) = 23/15$.

### A.8. Data collection/construction

In the Job Openings for Economists (JOE), a university advertises its $m$ openings in $n$ fields. Each field would be given average openings of $m/n$. For example, in 2000, the Department of Economics at Texas A&M University had five openings in 9 different fields. We assign each field with 5/9 openings. After the assignments for all American universities, the sum of all universities by each field is the total number of openings in that field.

To obtain the number of candidates for each field, we obtain the CV and/or the brief introduction for all job market candidates in top 50 economics departments in the US defined in Dusansky and Vernon (1998). We use the first field listed in each candidate's CV or in the brief introduction of the candidate. The total number of supplies in each field is the sum of all candidates in that field.

To obtain the total number of successful matches, we find out the new faculty members that each department hires in the fall semester of that year by searching the departmental websites and/or phone calls. To determine each newly hired faculty's field, we assign the field if the faculty's field is one of the advertised fields. If not, among the advertised fields, the field closest to the candidate's field would be chosen. In this case, the candidate's field would also be updated to the current field, so does the total number of candidates in each field. If a new hired faculty is not in the early list of candidates, the candidate is added in the list.

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